Factorial Difference-in-Differences

Yiqing Xu (Stanford)

UCLA CCPR

Anqi Zhao Peng Ding

(Duke) (Berkeley)

• Cao, Xu & Zhang (2022): "How social capital saved lives during China's Great Famine"

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- Data structure

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Social Capital: High

Social Capital: Low

- Cao, Xu & Zhang (2022): "How social capital saved lives during China's Great Famine"
- Data structure
 - A baseline factor G: time-invariant measure of social capital



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 - ► A baseline factor G: time-invariant measure of social capital
 - Event time: {Pre-Famine, Famine, Post-Famine}
- Estimation
 - Difference-in-differences (DID), or equivalently, two-way fixed effects (TWFE)



Outcome Means by Levels of Social Capital

Year



1964

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- Estimation
 - Difference-in-differences (DID), or equivalently, two-way fixed effects (TWFE)
- Interpretation
 - Descriptively: "the rise in the mortality rate during the famine years is significantly smaller in counties with higher social capital"
 - Causally: "we interpret these differences as the effects of social capital on famine relief."





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How Do Immigrants Respond to Discrimination? The Case of Germans in the US During World War I

VASILIKI FOUKA Stanford University

 \mathbf{T} study the effect of taste-based discrimination on the assimilation decisions of immigrant minorities. Do discriminated minority groups increase their assimilation efforts in order to avoid discrimination and public harassment or do they become alienated and retreat in their own communities? I exploit an exogenous shock to native attitudes, anti-Germanism in the United States during World War I, to empirically identify the reactions of German immigrants to increased native hostility. I use two measures of assimilation efforts: naming patterns and petitions for naturalization. In the face of increased discrimination, Germans increase their assimilation investments by Americanizing their own and their children's names and filing more petitions for US citizenship. These responses are stronger in states that registered higher levels of anti-German hostility, as measured by voting patterns and incidents of violence against Germans.

> American Economic Review 2020, 110(11): 3454–3491 https://doi.org/10.1257/aer.20191054

Devotion and Development: Religiosity, Education, and Economic Progress in Nineteenth-Century France[†]

By MARA P. SQUICCIARINI*

This paper studies when religion can hamper diffusion of knowledge and economic development, and through which mechanism. I examine Catholicism in France during the Second Industrial Revolution (1870–1914). In this period, technology became skill-intensive, leading to the introduction of technical education in primary schools. I find that more religious locations had lower economic development after 1870. Schooling appears to be the key mechanism: more religious areas saw a slower adoption of the technical curriculum and a push for religious education. In turn, religious education was negatively associated with industrial development 10 to 15 years later, when schoolchildren entered the labor market. (*JEL* D83, I21, I26, N33, Z12)

ARTICLE



From powerholders to stakeholders: State-building with elite compensation in early medieval China 😳

Joy Chen¹ | Erik H. Wang² | Xiaoming Zhang³

¹School of Economics, Renmin University of China, Beijing, China

²Wilf Family Department of Politics, New York University, New York, New York, USA

Abstract

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Authors contributed equally to this research and therefore share first-authorship equally. As far as "corresponding authorship" is concerned, each author is the corresponding author

How do rulers soften resistance by local powerholders to state-building efforts? This paper highlights a strategy of compensation, where elites receive government offices in exchange for relinquishing their localist interests, and become uprooted and integrated into the national political system as stakeholders. We explore this strategy in the context of the Northern Wei Dynasty of China (386–534 CE) that terminated an era of state weakness during which aristocrats exercised local autonomy through strongholds. Exploiting a comprehensive state-building reform in the late fifth century, we find that aristocrats from previously autonomous localities were disproportionately recruited into the bureaucracy as compensation for accepting stronger state presence. Three mechanisms of bureaucratic compensation facilitated statebuilding. Offices received by those aristocrats: (1) carried direct benefits, (2) realigned their interests toward the ruler, and (3) mitigated credible commitment problems. Our findings shed light on the "First Great Divergence" between Late Antiquity Europe and Medieval China.

EXPLAINING OUT-GROUP BIAS IN WEAK STATES Religion and Legibility in the 1891/1892 Russian Famine

By VOLHA CHARNYSH 💿

Department of Political Science, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA. E-mail: charnysh@mit.edu

ABSTRACT

Two dominant explanations for ethnic bias in distributional outcomes are electoral incentives and out-group prejudice. This article proposes a novel and complementary explanation for the phenomenon: variation in legibility across ethnic groups. The author argues that states will allocate fewer resources to groups from which they cannot gather accurate information or collect taxes. The argument is supported by original data on state aid from the 1891/1892 famine in the Russian Empire. Qualitative and quantitative analyses show that districts with a larger Muslim population experienced higher famine mortality and received less generous public assistance. The Muslims, historically ruled via religious intermediaries, were less legible to state officials and generated lower fiscal revenues. State officials could not count on the repayment of food loans or collect tax arrears from Muslim communes, so they were more likely to withhold aid. State relief did not vary with the presence of other minorities that were more legible and generated more revenue.

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Discrimination against German immigrants X World War I Do

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Muslim Share \times 1891/1892 Russia Famine

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THE POTATO'S CONTRIBUTION TO POPULATION AND URBANIZATION: EVIDENCE FROM A HISTORICAL EXPERIMENT*

NATHAN NUNN AND NANCY QIAN

We exploit regional variation in suitability for cultivating potatoes, together with time variation arising from their introduction to the Old World from the Americas, to estimate the impact of potatoes on Old World population and urbanization. Our results show that the introduction of the potato was responsible for a significant portion of the increase in population and urbaniza-

Same DID Estimator, A Different Research Design

Econometrica, Vol. 84, No. 2 (March, 2016), 677–733

ELITE RECRUITMENT AND POLITICAL STABILITY: THE IMPACT OF THE ABOLITION OF CHINA'S CIVIL SERVICE EXAM

BY YING BAI AND RUIXUE JIA¹

This paper studies how the abolition of an elite recruitment system—China's civil exam system that lasted over 1,300 years—affects political stability. Employing a panel data set across 262 prefectures and exploring the variations in the quotas on the entrylevel exam candidates, we find that higher quotas per capita were associated with a higher probability of revolution participation after the abolition and a higher incidence of uprisings in 1911 that marked the end of the 2,000 years of imperial rule. This finding is robust to various checks including using the number of small rivers and short-run exam performance before the quota system as instruments. The patterns in the data appear most consistent with the interpretation that in regions with higher quotas per capita under the exam system, more would-be elites were negatively affected by the abolition. In addition, we document that modern human capital in the form of those studying in Japan also contributed to the revolution and that social capital strengthened the effect of quotas on revolution participation.

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American Economic Review

• Factorial

classic topic in statistics

► factorial experiments pioneered by R. A. Fisher and F. Yates

two treatment factors: their main effects and interaction are of interest

Same DID Estimator, A Different Research Design "Factorial Difference-in-Differences"

CHINA'S CIVIL SERVICE EXAM

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Same DID Estimator, A Different Research Design "Factorial Difference-in-Differences"

Difference-in-differences

- popular in economics and related fields
- ► a "research design" for causal inference with observational data
- leverage panel data (units × times) to identify causal effects and the revolution and that social capital strengthened























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- Under no anticipation & parallel trends, the DID <u>estimator</u> identifies treatment effect heterogeneity of the event



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This Paper

Setting, Estimand, Estimator, ID Assumptions & ID Results

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- Identifying G's causal effect requires stronger assumptions



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- Under no anticipation & parallel trends, the DID <u>estimator</u> identifies treatment effect heterogeneity of the event
 - Not causal
 - ► Need additional analytical tools (factorial designs) to clarify
 - With covariates, common TWFE models need modification
- Identifying G's causal effect requires stronger assumptions
- Factorial DID includes canonical DID as a special case with an additional assumption



Related Literature

- DID and TWFE
 - "Regression DD": Card (1992); Angrist & Pischke (2009); Shahn & Hatfield (2024)
 - ► For reviews of recent development: Roth et al. (2023); Chiu et al. (2023); Arkhangelsky and Imbens (2023)
- Factorial designs
 - VanderWeele (2009); Dasgupta et al (2015); Bansak (2020); Zhao and Ding (2021)
- Bartik instruments & shift shares (e.g., local industry share x common temporal shock)
 - ▶ e.g. Paul Goldsmith-Pinkham et al. (2020); Borusyak, Hull & Jaravel (2022)
- Lord's paradox
 - Lord (1967); Holland and Rubin (1986)

Roadmap

- Motivation
- Setup & Estimands
- Identification
- Extensions
- Example: Clans and Calamity

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Two-group, two-periods; no covariates

Motivation



Two-group, two-periods; no covariates

Motivation



Two-group, two-periods; no covariates

- Study population: $i = 1, 2, \dots, n$
- Timing of the event is fixed
- Time periods: t = pre, post
- Baseline factor: $G_i \in \{0,1\}$
- Data: { G_i , $Y_{i,pre}$, $Y_{i,post}$: $i = 1, 2, \dots, n$ }



Two-group, two-periods; no covariates

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- Exposure to the event in the post period: $Z_i = 1$, $i = 1, 2, \dots, n$



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Factorial DID



Canonical DID

Two-group, two-periods; no covariates

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Factorial DID



Canonical DID

	Pre-Event	Post-Eve
$G_{i} = 1$	$\frac{1}{n_1} \sum_{i:G_i=1} Y_{i,\text{pre}}$	$\frac{1}{n_1} \sum_{i:G_i=1}^{N_1} $
$G_i = 0$	$\frac{1}{n_0} \sum_{i:G_i=0} Y_{i,\text{pre}}$	$\frac{1}{n_0} \sum_{i:G_i=0}^{N_0} \frac{1}{n_0} \sum_{i:G_i=0}^{N_0} $



	Pre-Event	Post-Eve
$G_{i} = 1$	$\frac{1}{n_1} \sum_{i:G_i=1} Y_{i,\text{pre}}$	$\frac{1}{n_1} \sum_{i:G_i=1}^{N_1} $
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Define



	Pre-Event	Post-Eve
$G_{i} = 1$	$\frac{1}{n_1} \sum_{i:G_i=1} Y_{i,\text{pre}}$	$\frac{1}{n_1} \sum_{i:G_i=1}^{N_1} $
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Define

•
$$\hat{\tau}_{\text{DID}} = \frac{1}{n_1} \sum_{i:G_i=1} (Y_{i,\text{post}} - Y_{i,\text{pre}}) - \frac{1}{n_0} \sum_{i:G_i=0} (Y_{i,\text{post}} - Y_{i,\text{pre}})$$



	Pre-Event	Post-Eve
$G_{i} = 1$	$\frac{1}{n_1} \sum_{i:G_i=1} Y_{i,\text{pre}}$	$\frac{1}{n_1} \sum_{i:G_i=1}^{N_1} $
$G_i = 0$	$\frac{1}{n_0} \sum_{i:G_i=0} Y_{i,\text{pre}}$	$\frac{1}{n_0} \sum_{i:G_i=0}^{N_0} \frac{1}{n_0} \sum_{i:G_i=0}^{N_0} $

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Focus on $\tau_{\text{DID}} = \text{plim } \hat{\tau}_{\text{DID}} = \mathbb{E}[\Delta Y_i | G_i = 1] - \mathbb{E}[\Delta Y_i | G_i = 0]$

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• Recall: in observed data, $Z_i = 1$ for all units

- Individual conditional effect
 - $\tau_{i,Z|G=g} = Y_{i,post}(g,1) Y_{i,post}(g,0)$

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- Individual interaction effect
 - $\tau_{i,\text{inter}} = Y_{i,\text{post}}(1,1) Y_{i,\text{post}}(1,0) Y_{i,\text{post}}(0,1) + Y_{i,\text{post}}(0,0)$

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- Average causal interaction (VanderWeele, 2009; Bansak, 2020)
 - $\tau_{\text{inter}} = \mathbb{E}[Y_{i,\text{post}}(1,1) Y_{i,\text{post}}(1,0) Y_{i,\text{post}}(0,1) + Y_{i,\text{post}}(0,0)]$





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 - $\tau_{\text{em}} = \mathbb{E}[\tau_{i,Z|G=1} \mid G_i = 1] \mathbb{E}[\tau_{i,Z|G=0} \mid G_i = 0]$





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Roadmap

- Motivation
- Setup & Estimands
- Identification
- Extensions
- Example: Clans and Calamity



A statistical estimand consistently estimated by $\hat{\tau}_{\rm DID}$



A statistical estimand consistently estimated by $\hat{\tau}_{\rm DID}$

An associative estimand describing effect heterogeneity



No anticipation & Parallel trends



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 τ_{inter}

A causal estimand targeting causal moderation

$\tau_{\rm att}$

A causal estimand targeting the effect of Z for G = 1

 $\tau_{G|Z=1}$







No Anticipation

 $Y_{i,\text{pre}}(g,0) = Y_{i,\text{pre}}(g,1)$ for all *i* and g = 0,1



No Anticipation

 $Y_{i,\text{pre}}(g,0) = Y_{i,\text{pre}}(g,1)$ for all i and g = 0,1



No Anticipation

$$Y_{i,\text{pre}}(g,0) = Y_{i,\text{pre}}(g,1)$$
 for all i and $g = 0,1$

Parallel Trends

 $\mathbb{E}[\Delta Y_{i}(1,0) \mid G_{i}=1] = \mathbb{E}[\Delta Y_{i}(0,0) \mid G_{i}=0]$



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$$Y_{i,\text{pre}}(g,0) = Y_{i,\text{pre}}(g,1)$$
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No Anticipation

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 for all i and $g = 0,1$

Parallel Trends

 $\mathbb{E}[\Delta Y_{i}(1,0) \mid G_{i}=1] = \mathbb{E}[\Delta Y_{i}(0,0) \mid G_{i}=0]$

Proposition

Under no anticipation and parallel trends,

$$\tau_{\text{DID}} = \tau_{\text{em}} = \mathbb{E}[\tau_{i,Z|G=1} \mid G_i = 1] - \mathbb{E}[\tau_{i,Z|G=0} \mid G_i = 0] .$$



No anticipation & Parallel trends



A statistical estimand consistently estimated by $\hat{\tau}_{\mathrm{DID}}$

An associative estimand describing effect heterogeneity

> A causal estimand targeting the effect of Z for G = 1

au_{em}



A causal estimand targeting causal moderation

$au_{\rm att}$

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Factorial DID



Factorial DID



Motivation

 $\tau_{\text{DID}} = \tau_{\text{em}} = \mathbb{E}[Y_{i,\text{post}}(1,1) - Y_{i,\text{post}}(1,0) \mid G_i = 1]$ $-\mathbb{E}[Y_{i,\text{post}}(0,1) - Y_{i,\text{post}}(0,0) \mid G_i = 0]$

Factorial DID



Motivation

$$Y_{i,\text{post}}(1,1) - Y_{i,\text{post}}(1,0) \mid G_i = 1$$

 $-\mathbb{E}[Y_{i,\text{post}}(0,1) - Y_{i,\text{post}}(0,0) \mid G_i = 0]$

Exclusion Restriction on Z

 $Y_{i,\text{post}}(0,1) = Y_{i,\text{post}}(0,0)$ for all units with $G_i = 0$

Factorial DID



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Factorial DID \rightarrow Canonical DID



$$Y_{i,\text{post}}(1,1) - Y_{i,\text{post}}(1,0) \mid G_i = 1$$

Exclusion Restriction on Z

 $Y_{i,\text{post}}(0,1) = Y_{i,\text{post}}(0,0)$ for all units with $G_i = 0$

Proposition

Under no anticipation, parallel trends, and the

exclusion restriction, $\tau_{\rm DID}=\tau_{\rm att}$.



No anticipation & Parallel trends



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A causal estimand targeting causal moderation







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Exclusion restriction on Z for G = 0



au_{att}

A causal estimand targeting the effect of Z for G = 1



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 $\tau_{G|Z=1}$



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Exclusion restriction on Z for G = 0

au_{em}

au_{att}

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• Imagine an unobservable U that determines how units respond to the event



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Why $\tau_{\rm DID}$ May Not Identify Causal Moderation under Parallel Trends?



- Imagine an unobservable U that determines how units respond to the event
- $U \mod be$ correlated with G

Why $\tau_{\rm DID}$ May Not Identify Causal Moderation under Parallel Trends?



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Why $\tau_{\rm DID}$ May Not Identify Causal Moderation under Parallel Trends?



- \bullet Imagine an unobservable U that determines how units respond to the event
- $U \mod be$ correlated with G
- Therefore, $\tau_{\rm DID}$ cannot be interpreted as the causal moderation of G

Identifying Causal Interaction (Causal Moderation)



Motivation

 $\tau_{\text{DID}} = \tau_{\text{em}} = \mathbb{E}[Y_{i,\text{post}}(1,1) - Y_{i,\text{post}}(1,0) \mid G_i = 1]$

 $-\mathbb{E}[Y_{i,\text{post}}(0,1) - Y_{i,\text{post}}(0,0) \mid G_i = 0]$


Motivation

 $-\mathbb{E}[Y_{i,\text{post}}(0,1) - Y_{i,\text{post}}(0,0) \mid G_i = 0]$

 $\tau_{\text{inter}} = \mathbb{E}[Y_{i,\text{post}}(1,1) - Y_{i,\text{post}}(1,0)]$ $-\mathbb{E}[Y_{i,\text{post}}(0,1) - Y_{i,\text{post}}(0,0)]$



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Sufficient condition: $\Delta Y_i(g, z) \perp G_i$



Motivation



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 $\tau_{\text{DID}} = \tau_{\text{em}} = \mathbb{E}[Y_{i,\text{post}}(1,1) - Y_{i,\text{post}}(1,0) \mid G_i = 1] \qquad \qquad \boxed{2} \qquad \tau_{\text{inter}} = \mathbb{E}[Y_{i,\text{post}}(1,1) - Y_{i,\text{post}}(1,0)] \\ -\mathbb{E}[Y_{i,\text{post}}(0,1) - Y_{i,\text{post}}(0,0) \mid G_i = 0] \qquad \qquad -\mathbb{E}[Y_{i,\text{post}}(0,1) - Y_{i,\text{post}}(0,0)]$

Sufficient condition: $\Delta Y_i(g, z) \perp G_i$

(Orthogonality between G and first-differenced potential outcomes)

Weaker than (quasi-)random assignment: $Y_{i,t}(g,z) \perp G_i$





Post



 $\tau_{\text{DID}} = \tau_{\text{em}} = \mathbb{E}[Y_{i,\text{post}}(1,1) - Y_{i,\text{post}}(1,0) \mid G_i = 1] \qquad \qquad \boxed{2} \qquad \tau_{\text{inter}} = \mathbb{E}[Y_{i,\text{post}}(1,1) - Y_{i,\text{post}}(1,0)] \\ -\mathbb{E}[Y_{i,\text{post}}(0,1) - Y_{i,\text{post}}(0,0) \mid G_i = 0] \qquad \qquad -\mathbb{E}[Y_{i,\text{post}}(0,1) - Y_{i,\text{post}}(0,0)]$

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Post

 \Rightarrow



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(Orthogonality between G and first-differenced potential outcomes)

Weaker than (quasi-)random assignment: $Y_{i,t}(g, z) \perp G_i$

$$\tau_{\text{inter}} = \mathbb{E}[Y_{i,\text{post}}(1,1) - Y_{i,\text{post}}(0,1)] = \tau_{G|Z=1}$$

Summary of identification Results

No anticipation & Parallel trends



A statistical estimand consistently estimated by $\hat{\tau}_{\mathrm{DID}}$

An associative estimand describing effect heterogeneity

Exclusion restriction on Z for G = 0



$\tau_{\rm att}$

A causal estimand targeting the effect of Z for G = 1



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 $\Delta Y(g,z) \perp G$

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Canonical DID (original)

Define <u>canonical DID research design</u> as the combination of:

- 2×2 data structure
- Identification results:
 - Under no anticipation & parallel trends, $\hat{\tau}_{\mathrm{DID}}$ identifies τ_{att}



Canonical DID (reframed)

Define <u>canonical DID</u> research design as the combination of:

- 2×2 data structure & universal exposure
- Identification results:
 - Under no anticipation & parallel trends & exclusion restriction on Z, $\hat{\tau}_{\text{DID}}$ identifies τ_{att}



Canonical DID (reframed)

Define <u>canonical DID research design</u> as the combination of:

- 2×2 data structure & universal exposure
- Identification results:
 - Under no anticipation & parallel trends & exclusion restriction on Z, $\hat{\tau}_{\text{DID}}$ identifies τ_{att}

Factorial DID

Define <u>factorial DID</u> research design as the combination of:

- 2×2 data structure & universal exposure
- Identification results:
 - 1. Under no anticipation & parallel trends, $\hat{\tau}_{\text{DID}}$ identifies τ_{em}
 - 2. Under no anticipation & parallel trends & $\Delta Y_i(g,z) \perp G_i$, $\hat{\tau}_{\text{DID}}$ identifies τ_{inter}
 - 3. Under no anticipation & parallel trends & $\Delta Y_i(g,z) \perp G_i$ & exclusion restriction on G, $\hat{\tau}_{\text{DID}}$ identifies $\tau_{G|Z=1}$



"A large university is interested in investigating the effects on the students of the diet provided in the university dining halls and any sex differences in these effects ... [t]he weight of each student at the time of his (/her) arrival in September and his weight the following June are recorded." (Lord 1967, p. 304)

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- Statistician 2: larger effect on male students (6.34) from: \bullet $Y_{i,\text{post}} \sim 1 + male_i + Y_{i,\text{pre}}$





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- Resolution
 - Holland and Rubin (1986): Untestable assumptions on $Y_{i,post}(g,0)$
 - Statistician 1: $Y_{i,post}(g,0) = Y_{i,pre}$
 - Statistician 2: $Y_{i,post}(g,0) = \beta Y_{i,pre} + \gamma_g$
 - Statistician 3 (Factorial DID): Under no anticipation & parallel trends, $\tau_{em} = \tau_{\rm DID} = 0$





Roadmap

- Motivation
- Setup & Estimands
- Identification
- Extensions
 - Conditionally valid assumptions
 - *Multiple pre- and post- periods
 - ► *Multi-valued G
- Example: Clans and Calamity

• Researchers often run the following TWFE regression:

 $Y_{it} = \alpha_i + \xi_t + \tau G_i \cdot \text{Post}_t + \beta^T \mathbf{X}_i \cdot \frac{\text{Post}_t}{1} + \epsilon_{it}$

• Researchers often run the following TWFE regression:

$$Y_{it} = \alpha_i + \xi_t + \tau G_i \cdot \text{Post}_t -$$

• Believe $\Delta Y_i(g,z) \perp G_i \mid X_i$ is more plausible than $\Delta Y_i(g,z) \perp G_i$

 $+ \beta^{\mathrm{T}} \mathbf{X}_{\mathrm{i}} \cdot \mathrm{Post}_{\mathrm{t}} + \epsilon_{\mathrm{it}}$

• Researchers often run the following TWFE regression:

$$Y_{it} = \alpha_i + \xi_t + \tau G_i \cdot \text{Post}_t - \tau G_i - \tau G_i \cdot \text{Post}_t - \tau G_i -$$

- Believe $\Delta Y_i(g, z) \perp G_i \mid X_i$ is more plausible than $\Delta Y_i(g, z) \perp G_i$
- Simple improvements: (a) demean X_i ; (b) add interaction terms

$$Y_{it} = \alpha_i + \xi_t + \tau G_i \cdot \text{Post}_t + \beta^T \mathbf{X}$$

 $+ \beta^{\mathrm{T}} \mathbf{X}_{\mathrm{i}} \cdot \mathbf{Post}_{\mathrm{t}} + \epsilon_{\mathrm{it}}$

 $X_i \cdot Post_t + \gamma^T G_i \cdot X_i \cdot Post_t + \epsilon_{it}$

• Researchers often run the following TWFE regression:

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$$Y_{it} = \alpha_i + \xi_t + \tau G_i \cdot \text{Post}_t + \beta^T \mathbf{X}$$

• Can leverage more flexible models of ΔY_i on X_i for subgroups $G_i = 1,0$

 $+ \beta^{\mathrm{T}} \mathbf{X}_{\mathrm{i}} \cdot \mathrm{Post}_{\mathrm{t}} + \epsilon_{\mathrm{it}}$

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- Can leverage more flexible models of ΔY_i on X_i for subgroups $G_i = 1,0$
 - Transform data into wide form; replace Y_i with ΔY_i

 $+ \beta^{\mathrm{T}} \mathbf{X}_{\mathrm{i}} \cdot \mathrm{Post}_{\mathrm{t}} + \epsilon_{\mathrm{it}}$

 $\mathbf{X}_{i} \cdot \mathbf{Post}_{t} + \gamma^{T} \mathbf{G}_{i} \cdot \mathbf{X}_{i} \cdot \mathbf{Post}_{t} + \epsilon_{it}$

• Researchers often run the following TWFE regression:

$$Y_{it} = \alpha_i + \xi_t + \tau G_i \cdot \text{Post}_t -$$

- Believe $\Delta Y_i(g,z) \perp G_i \mid X_i$ is more plausible than $\Delta Y_i(g,z) \perp G_i$
- Simple improvements: (a) demean X_i ; (b) add interaction terms

$$Y_{it} = \alpha_i + \xi_t + \tau G_i \cdot \text{Post}_t + \beta^T \mathbf{X}$$

- Can leverage more flexible models of ΔY_i on X_i for subgroups $G_i = 1,0$
 - Transform data into wide form; replace Y_i with ΔY_i
 - Apply a variety of estimators developed for selection-on-observables designs (e.g., stratification, matching, balancing, IPW, AIPW, outcome modeling, double machine learning...)

 $+ \beta^{T} X_{i} \cdot Post_{t} + \epsilon_{it}$

 $X_i \cdot Post_t + \gamma^T G_i \cdot X_i \cdot Post_t + \epsilon_{it}$





Motivation



Lesson 1: Pretrend tests can help assess the parallel trends assumption, but not $\Delta Y(g,z) \perp G$ ${ \bullet }$



- Lesson 1: Pretrend tests can help assess the parallel trends assumption, but not $\Delta Y(g,z) \perp G$

<u>Lesson 2</u>: Using post-periods as non-event periods requires an additional "no carryover effect" assumption

Roadmap

- Motivation
- Setup & Estimands
- Identification
- Extensions
- Example: Clans and Calamity

Example: Clans and Calamity

Motivation

Example: Clans and Calamity

• Event — The Great famine (1958-1961)

Example: Clans and Calamity

- Event The Great famine (1958-1961)
- G Social Capital (proxied by genealogies)
 - ► No Genealogies: 412 counties
 - ► Have pre-PRC genealogies: 509 counties

Raw Data: Group Means

Motivation

Year
Conditional on Covariates

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Difference-in-Differences

Year



Two-way Fixed Effects with Covariates





Estimated Coefficient (w/ 95% CI)

Two-way Fixed Effects with Additional Interaction Terms

Year



Motivation

Augmented Inverse Propensity Score Weighting

Year

Multi-valued G



Multi-valued G



#Genealogies per 10,000 people (sqrt scale)

Multi-valued G — AIPW Estimator

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Group 0 vs Group 1



Multi-valued G — AIPW Estimator

Group 0 vs Group 1





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