17.802 – Recitation 1

Potential Outcomes Model

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Notations

- Potential outcomes: $\{Y_{0i}, Y_{1i}\}$
- Realization: Y_i
- Another way to write it: $Y_i = D_i \cdot Y_{1i} + (1 - D_i) \cdot Y_{0i}$
- Meaning $Y_i = \begin{cases} Y_{1i} & \text{if } D_i = 1 \\ Y_{0i} & \text{if } D_i = 0 \end{cases}$
- Causal effect: $\alpha_i = Y_{1i} - Y_{0i}$
- We only observe Y_{0i} or Y_{1i}

Stable Unit Treatment Value Assumption (SUTVA)

- The impact of treating a unit does NOT depend on how the treatments assigned
- SUTVA assumes:
  - No interference: the treatment status of one unit does not affect the potential outcomes of another
  - No variation in treatment: the treatments for all units are comparable Potential outcomes are thought to be fixed for each individual
- Violations:
  - Vaccination (interference)
  - Fertilizer A and B on crop yield, but each fertilizer has a lot of versions (variation in treatment)
  - Hawthorne effect: people change their behavior if they know they are being observed (variation in treatment)

The case of interference

- Potential outcomes

$$Y_{1i}(\mathbf{D}) = \begin{cases} Y_{1i}(1,1) \\ Y_{1i}(1,0) \end{cases} \quad Y_{0i}(\mathbf{D}) = \begin{cases} Y_{0i}(0,1) \\ Y_{0i}(0,0) \end{cases}$$

- Causal effect(s)

$$\alpha_i(\mathbf{D}) = \begin{cases} Y_{1i}(1,1) - Y_{0i}(0,0) \\ Y_{1i}(1,1) - Y_{0i}(0,1) \\ Y_{1i}(1,0) - Y_{0i}(0,0) \\ Y_{1i}(1,0) - Y_{0i}(0,1) \\ Y_{1i}(1,1) - Y_{1i}(1,0) \\ Y_{0i}(0,1) - Y_{0i}(0,0) \end{cases}$$

ATE and ATT

- Average treatment effect

$$\alpha_{ATE} = \mathbf{E}[Y_{1i} - Y_{0i}]$$

- Average treatment effect on the treated

$$\alpha_{ATT} = \mathbf{E}[Y_{1i} - Y_{0i} | D_i = 1]$$

- Average treatment effect on the controls

$$\alpha_{ATC} = \mathbf{E}[Y_{1i} - Y_{0i} | D_i = 0]$$

Selection Bias

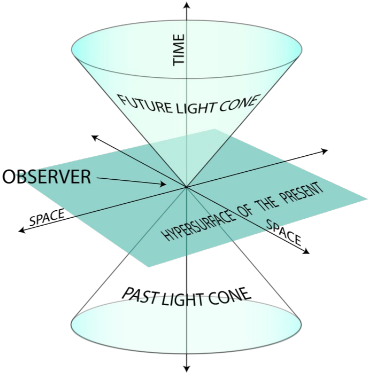
- Math

$$\mathbf{E}[Y|D=1]-\mathbf{E}[Y_i|D_i=0]=\mathbf{E}[Y_{1i}|D_i=1]-\mathbf{E}[Y_{0i}|D_i=0]$$
$$=\underbrace{\mathbf{E}[Y_{1i}-Y_{0i}|D_i=1]}_{\text{ATT}}+\underbrace{\{\mathbf{E}[Y_{i0}|D_i=1]-\mathbf{E}[Y_{0i}|D_i=0]\}}_{\text{BIAS}}$$

- Example: the effect of MIT education

Causality in "More Easy Physical Sciences"

"When signal is not faster than light (assumption), a Lorentz transformation of reference frame such that event A ("cause") and B ("effect") is simultaneous is not possible."



Causality is possible when the speed of light is constant

Rubin's Model

- Causality is about time, but Rubin points out:
- The effect of a cause is always relative to another cause
- $\{Y_{0i}, Y_{1i}\}$
- The treatment causes the effect $\{Y_{1i} - Y_{0i}\}$
- Fundamental problem of causal inference:
  i.e., cannot observe both Y_{0i} and Y_{1i}
- Two solutions: *scientific* and *statistical*
- A *scientific* solution exploits homogeneity or invariance assumptions
  e.g., a rock is a rock
- A *statistical* solution makes use of the population
  e.g., $T = E(Y_1) - E(Y_0)$

Special Cases

- Temporal stability: sequential exposure of i to control and treatment
- Unit homogeneity: perfect lab experiment, $Y_{0i} = Y_{0j}$, $Y_{1i} = Y_{1j}$
- **Independence**
- Constant effect

What Can be a Cause?

(A) She did well on the exam because she is a woman.
(B) She did well on the exam because she studied for it.
(C) She did well on the exam because she was coached by her teacher.

- (C): Had she not been coached by her teacher? Check
- (A): Had she not been a woman?

  Kempthorne (1978): "It is epistemological nonsense to talk about one trait of an individual causing or determining another trait of the individual."
- (B): Was she studying for it voluntary?
- The general problem is in deciding when it is an attribute of units and when it is a cause that can act on units

Messages in Holland (1986)

- Study the effect of a cause rather than cause of a given effect
- Effects of causes are relative to other causes
- Attributes cannot be causes

  Rubin disagrees: attributes can be causes under Fisher's Null Hypothesis
  (then SUTVA is automatically satisfied)

Comments and Responses

Rubin's comment (Important)

- Always think about SUTVA carefully
- No causation without manipulation

Cox's comment

- The distinction between quasitreatment and intrinsic variables is a matter of viewpoint
- Different "layers" of interpretation, e.g., turning a light switch

Glymour's comment

- The counterfactual needs to be well-defined
- Some of the restrictions are unwarranted

Granger's comment

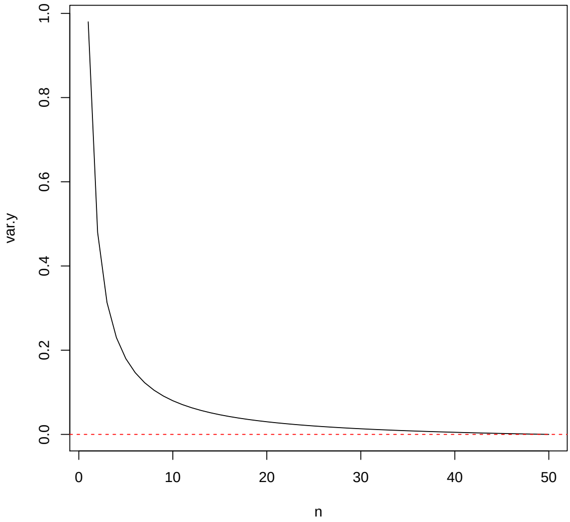
- Experimental framework may not be relevant for testing temporal causality

Question 1c

- A collection of fixed numbers $\{y_1, y_2, \cdots, y_N\}$ of unknown distribution
- Each has an indicator $\{z_1, z_2, \cdots, z_N\}$, $z_i \in \{0, 1\}$
- We are interested in $\mu_y = \frac{1}{N}\sum_{i=1}^{N} y_i$, which is a also number
- But we can only observe $\{z_1 y_1, z_2 y_2, \cdots, z_N y_N\}$
- What is our best shot?

- What about $\tilde{y}_i = \frac{\sum_{i=1}^{N} z_i y_i}{\sum_{i=1}^{N} z_i}$?
- $var(\tilde{y}) = (1 - \frac{n}{N})\frac{\sigma_y^2}{n}$

$var(\tilde{y})$



Question 1c

The variance of $\tilde{y}$ is:

$$var(\tilde{y}) = var\left(\frac{1}{n}\sum_{i=1}^{N} z_i y_i\right) = \frac{1}{n^2}\left[\sum_{i=1}^{N} y_i^2 var(z_i) + \sum_{i=1}^{N}\sum_{j\neq i} y_i y_j cov(z_i, z_j)\right]$$

Show that:

$$cov(z_i, z_j) = \frac{-n(N-n)}{N^2(N-1)} \quad \text{and}$$

$$var(\tilde{y}) = \frac{1}{n^2}\frac{n}{N}(1-\frac{n}{N})\left[\sum_{i=1}^{N} y_i^2 - \frac{1}{N-1}\sum_{i=1}^{N}\sum_{j\neq i} y_i y_j\right]$$